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FORCED TRANSVERSE VIBRATIONS OF A SOLID, ELASTIC CORE CASE-BONDED TO AN INFINITELY-LONG, RIGID CYLINDER

by

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AUGUST 1961

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Department of Civil Engineering

The Catholic University of America

Washington 17, D.C.

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Forced Transverse Vibrations of a Solid, Elastic Core Case-Bonded to an Infinitely-Long, Rigid Cylinder

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Technical Report No. 1
To the
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August 1961

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Abstract

The problem is solved of the forced, transverse vibrations of a solid, compressible, elastic core case-bonded to an infinitely-long, rigid cylinder. It is shown that the ratio of the amplitude response of the core axis to the amplitude of the casing depends on both the frequency of the forced vibration and Poisson's ratio for the core material. By plotting amplitude ratio versus frequency curves for different values of Poisson's ratio it is demonstrated that the amplitude ratio versus frequency plot for an incompressible, elastic core is a simple line spectrum. On the basis of this result it is concluded that considerable care must be exercised when interpreting the results of solutions of problems wherein the assumption of incompressible material is involved.

Table of Contents

I. INTRODUCTION

The problem of the free, transverse vibrations of a solid, compressible, elastic core case-bonded to an infinitely-long, rigid, circular-cylindrical tank has been previously treated,* and the following frequency equations among other things were derived.

$$J_{n-1}(\Omega)J_{n+1}(k\Omega)+J_{n+1}(\Omega)J_{n-1}(k\Omega)=0,\quad (n=1,2,\ldots) \tag{1}$$

These transcendental frequency equations define a doubly infinite set of natural circular frequency coefficients Ω_{nm}. The mode of vibration is identified by the subscript n while the subscript m identifies the frequency number within a given mode. In Eq. (1) k^2 denotes the ratio of the square of the shear wave velocity c_s to the square of the dilatational wave velocity c_c; i. e.,

$$c_s^2 = \frac{G}{\rho} \tag{2a}$$

$$c_c^2 = \frac{2(1-\nu)}{1-2\nu}\left(\frac{G}{\rho}\right) \tag{2b}$$

$$k^2 = \frac{c_s^2}{c_c^2} = \frac{1-2\nu}{2(1-\nu)} \tag{2c}$$

Since k depends only on Poisson's ratio, it is clear that the natural frequencies of free vibrations, as defined by Eq. (1), also depend only on Poisson's ratio. Figure 1 has been reproduced from the previous paper* in order to demonstrate this dependency for the first-order (n = 1) natural frequency coefficients Ω_{1m}.

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* Baltrukonis, J. H., "Free Transverse Vibrations of a Solid Elastic Mass in an Infinitely-Long, Rigid, Circular-Cylindrical Tank" J. Appl. Mechanics 27 663 (December 1960)

These curves exhibit very peculiar and interesting shapes but we wish to make the point here that as k tends to zero, the curves tend to finite real values of the natural frequency coefficients. The zero value for k corresponds to a value of 1/2 for Poisson's ratio ν which value defines incompressible material. Thus, Fig. 1 demonstrates that natural frequencies do exist for incompressible material.

We recall the problem under consideration. In the previous paper the question was posed: "Can natural frequencies exist?" The answer was affirmative even for incompressible material. The question immediately arises: "How can natural frequencies exist for incompressible material when it occupies the entire internal volume of the tank?" The vibration under consideration is one of plane strain in which there can be no displacement out of the plane of a cross-section; it has been shown, however, that free vibrations can exist in the modes under discussion. In the present report we are concerned with the explanation of this apparent contradiction.

We shall consider the problem of transverse vibrations of the solid, compressible, elastic core when the infinitely-long, rigid casing is oscillated by some external means with simple harmonic motion. The response of the core at a generic point r will be calculated. This result will be specialized to obtain the amplitude response of the core axis. The ratio of the amplitude response of the core axis to the amplitude of the casing will then be plotted as a function of the forcing frequency for various values of Poisson's ratio tending to 1/2. It will be seen that as Poisson's ratio tends to 1/2, that is, as the core material tends to become incompressible, the amplitude ratio-forcing frequency plots tend to a simple line spectrum. Clearly,

this type of frequency response is physically impossible. That this should be the case is not surprising since incompressible material is an hypothetical material which cannot exist in nature. Nevertheless, the assumption of such a material is quite regularly used in practice. The present report presents one difficulty arising from the use of such an hypothetical material.

II. NOTATION

r, θ, z	radial, circumferential and axial coordinate variables of polar cylindrical coordinates
t	time
u_r, u_θ, u_z	radial, circumferential and axial components of displacement
ϕ, ψ, χ	displacement of potentials
k	ratio of shear wave velocity to dilatational wave velocity
c_c	dilatational wave velocity
c_s	shear wave velocity
ν	Poisson's ratio
G	shear modulus
ρ	mass density
e	mean normal strain $= \frac{1}{3}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right)$
∇^2	Laplacian operator $= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$
ω	natural circular frequency
Ω	natural circular frequency coefficient
p	forcing circular frequency
λ	forcing circular frequency coefficient
b	radius of interface between elastic core and rigid tank
n	order number of vibration mode
m	frequency number within a given mode

J_n Bessel function of the first kind of order n

C_1, C_2 Constants

W Amplitude of vibration of the rigid casing

$\overline{W}$ Amplitude response of the core axis

A prime over a quantity denotes the ordinary derivative of the quantity with respect to its argument.

III. STATEMENT AND SOLUTION OF THE PROBLEM

It is not difficult to show that the field equations for the vibrations of compressible, elastic continua in polar, cylindrical coordinates may be reduced to the following three equations of motion:

$$\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{3}{1-2\nu}\frac{\partial e}{\partial r} = \frac{\rho}{G}\frac{\partial^2 u_r}{\partial t^2} \tag{3a}$$

$$\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{3}{1-2\nu}\frac{1}{r}\frac{\partial e}{\partial \theta} = \frac{\rho}{G}\frac{\partial^2 u_\theta}{\partial t^2} \tag{3b}$$

$$\nabla^2 u_z + \frac{3}{1-2\nu}\frac{\partial e}{\partial z} = \frac{\rho}{G}\frac{\partial^2 u_z}{\partial t^2} \tag{3c}$$

We seek solutions of these equations of motion in terms of three displacement potentials as follows:

$$u_r = \frac{\partial \phi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z} + \frac{1}{r}\frac{\partial \chi}{\partial \theta} \tag{4a}$$

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi}{\partial \theta \partial z} - \frac{\partial \chi}{\partial r} \tag{4b}$$

$$u_z = \frac{\partial \phi}{\partial z} - \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\psi \tag{4c}$$

It may be verified by direct substitution that Eqs. (3) are identically satisfied by these displacement components provided we take the displacement potentials as solutions of the following differential equations:

$$\nabla^2\phi = \frac{k^2\rho}{G}\frac{\partial^2\phi}{\partial t^2} \tag{5a}$$

$$\nabla^2(\psi, \chi) = \frac{\rho}{G}\frac{\partial^2}{\partial t^2}\left(\psi, \chi\right) \tag{5b}$$

We recognize these differential equations as wave equations, solutions of which are well-known.

We now apply this general theory to the problem under consideration for which the boundary conditions are:

$$u_r \Big|_{r=b} = We^{ipt}\cos\theta \tag{6a}$$

$$u_\theta \Big|_{r=b} = -We^{ipt}\sin\theta \tag{6b}$$

$$u_z \Big|_{r=b} = 0 \tag{6c}$$

It is readily apparent that these boundary conditions define a forced, sinusoidal oscillation of the rigid tank with arbitrary frequency p and with amplitude W in the $\theta=0$ direction.

We take a solution for the displacement potentials of the following form:

$$\phi = C_1 \, J_1 \left(k\lambda \frac{r}{b}\right) e^{ipt} \cos \theta \tag{7a}$$

$$\psi = 0 \tag{7b}$$

$$\chi = C_2 \, J_1 \left(\lambda \frac{r}{b}\right) e^{ipt} \sin \theta \tag{7c}$$

It may be verified by direct substitution that these potentials are solutions of Eqs. (5) provided we take

$$\lambda^2 = \frac{\rho p^2 b^2}{G}. \tag{7d}$$

Because of its relation to the forcing frequency p, λ will be referred to as the forcing frequency coefficient.

Substitution from Eqs. (7) into Eqs. (4) results in the following expressions for the displacements:

$$u_r = \left[C_1 \frac{k\lambda}{b} J_1' \left(k\lambda \frac{r}{b}\right) + \frac{C_2}{r} J_1 \left(\lambda \frac{r}{b}\right)\right] e^{ipt} \cos \theta \tag{8a}$$

$$u_\theta = -\left[\frac{C_1}{r} \, J_1\left(k\lambda \frac{r}{b}\right) + C_2 \frac{\lambda}{b} J_1' \left(\lambda \frac{r}{b}\right)\right] e^{ipt} \sin \theta \tag{8b}$$

$$u_z = 0 \tag{8c}$$

We must now evaluate the constants involved in the solution given above. To this end we substitute from Eqs. (8) into the boundary conditions given by Eqs. (6) to obtain the following **non-homogeneous** system of two linear algebraic equations in the unknown constants:

$$\begin{vmatrix} (k\lambda)\, J_1'\,(k\lambda) & J_1\,(\lambda) \\ J_1\,(k\lambda) & \lambda J_1'\,(\lambda) \end{vmatrix} \begin{vmatrix} C_1 \\ C_2 \end{vmatrix} = \begin{vmatrix} Wb \\ Wb \end{vmatrix} \tag{9a}$$

Such a system can have a consistent solution only if the determinant of the coefficients of the unknowns does not vanish. On expansion of this determinant we obtain

$$\Delta(\lambda) = -\frac{1}{2}\, k\, \lambda^2 \left[J_0\,(\lambda)\; J_2\,(k\lambda) + J_2\,(\lambda)\; J_0\,(k\lambda) \right] \tag{9b}$$

On comparison of this result with Eq. (1) we find that Eq. (9b) has identically the same form as the frequency equation given by Eq. (1) except that the natural circular frequency coefficient Ω is replaced by the forcing frequency coefficient λ; i. e.,

$$\Delta(\Omega_{1m}) = 0, \quad (m = 1, 2, 3, \ldots.) \tag{9c}$$

Thus, the determinant of the coefficients of the unknowns in Eq. (9a) will vanish whenever the forcing frequency coefficient λ is equal to the first-order natural circular frequency coefficient Ω_{1m}. As a result, we cannot expect to obtain a consistent solution of Eq. (9a) whenever $\lambda = \Omega_{1m}$. With this restriction in mind we proceed to obtain the following solution for the arbitrary constants:

$$C_1 = -Wb\,\Delta^{-1}(\lambda)\,\lambda J_2(\lambda) \tag{10a}$$

$$\lambda \neq \Omega_{1m}$$

$$C_2 = -Wb\,\Delta^{-1}(\lambda)(k\lambda)J_2(k\lambda) \tag{10b}$$

wherein we have made use of the recurrence relations for the Bessel functions.

Finally, in view of Eqs. (10), Eqs. (8) become

$$u_r = \tfrac{1}{2}Wk\,\lambda^2\Delta^{-1}(\lambda)\Big[J_2(\lambda)J_2(k\lambda\tfrac{r}{b}) - J_2(k\lambda)J_2(\lambda\tfrac{r}{b}) - J_0(\lambda\tfrac{r}{b})J_2(k\lambda) - J_2(\lambda)J_0(k\lambda\tfrac{r}{b})\Big]\,e^{ipt}\cos\theta \tag{11a}$$

$$u_\theta = \tfrac{1}{2}Wk\,\lambda^2\Delta^{-1}(\lambda)\Big[J_2(\lambda)J_2(k\lambda\tfrac{r}{b}) - J_2(k\lambda)J_2(\lambda\tfrac{r}{b}) + J_0(\lambda\tfrac{r}{b})J_2(k\lambda) + J_2(\lambda)J_0(k\lambda\tfrac{r}{b})\Big]\,e^{ipt}\sin\theta \tag{11b}$$

To obtain some insight into the physics of the problem let us investigate further the response of the axis. From Eqs. (11) it follows that

$$u_x\Big|_{\theta=0} = u_r\Big|_{\theta=0} = \tfrac{1}{2}Wk\,\lambda^2\Delta^{-1}(\lambda)\Big[J_2(\lambda)J_2(k\lambda\tfrac{r}{b}) - J_2(k\lambda)J_2(\lambda\tfrac{r}{b}) - J_0(\lambda\tfrac{r}{b})J_2(k\lambda) - J_2(\lambda)J_0(k\lambda\tfrac{r}{b})\Big]\,e^{ipt}$$

and so

$$u_x\Big|_{\substack{r=0\\ \theta=0}} = \overline{W}e^{ipt} = -\tfrac{1}{2}Wk\,\lambda^2\Delta^{-1}(\lambda)\Big[J_2(k\lambda) + J_2(\lambda)\Big]\,e^{ipt}$$

Finally, therefore, if we make use of Eq. (9b)

$$\frac{\overline{W}}{W} = \left[J_2(\lambda) + J_2(k\lambda)\right] \left[J_0(\lambda)J_2(k\lambda) + J_2(\lambda)J_0(k\lambda)\right]^{-1} \qquad (12)$$

This result defines an amplification factor for the axis of the core. We observe that this amplification factor depends not only on the forcing frequency but also on Poisson's ratio, since k depends on Poisson's ratio only.

IV. ANALYSIS OF RESULTS AND CONCLUSIONS

Figure 1 shows the variation with the velocity ratio k of the first-order circular frequency coefficients for free, transverse vibrations of a solid, compressible elastic core case-bonded to an infinitely-long, rigid, circular cylindrical tank. These plots demonstrate that there exist natural modes of vibration even for an incompressible core when $k = 0$. This fact seems rather curious on closer consideration since the mode of vibration under investigation is one of plane strain and there does not appear to be any way in which an incompressible core can be deformed. In an attempt to provide at least a partial explanation of this apparent anomaly Figs. 2 have been plotted in order to demonstrate the variation with Poisson's ratio of the amplitude ratio-frequency characteristics of the core axis making use of Eq. (12). Since we are principally interested in the amplitude response in the neighborhood of the natural frequencies, the forcing frequency was normalized with the fundamental first-mode frequency in Fig. 2(a) and with the second first-mode frequency in Fig. 2(b). We observe that as Poisson's ratio approaches 1/2, the resonance peaks tend to become sharper and sharper until, finally, for an incompressible material ($\nu = 1/2$) the frequency response is a simple impulse function and the entire characteristic curve is a discrete line spectrum as shown in Fig. 3.

It is clear that such a response characteristic is physically impossible for a real material. That this should be the case ought not to be surprising inasmuch as an incompressible material is an ideal material which, a priori, would not be expected to behave physically, exactly and entirely, as a real, nearly incompressible material. Although many physical characteristics of incompressible materials

have counterparts in compressible materials, other properties of incompressible materials as, e. g., the nature of the frequency response, are impossible of physical realization. This is an important observation since many analytical studies are being carried out on the basis of this assumption. In most cases assumption of an incompressible material simplifies analysis considerably but we must be aware of its limitations.

Another interesting and important observation concerning Figs. 2 is the tremendous sensitivity to Poisson's ratio exhibited by the response curves. Very small changes in Poisson's ratio in the neighborhood of 1/2 produce considerable changes in the shape of the curves. This is important since many materials are very nearly incompressible, notably many high polymers including solid propellant materials.

Summarizing, the foregoing points up the necessity for considerable care when analyses are performed on the assumption of an ideally incompressible material. In many respects such material exhibits the properties of real materials but it sometimes displays characteristics that cannot be reproduced in nature. Furthermore, when the material under study is nearly incompressible, the analysis must be performed with considerable care and precision since such materials are very sensitive to small changes in Poisson's ratio. It is clear that we must take great care in conducting experimental measurements of Poisson's ratio for these nearly incompressible materials.

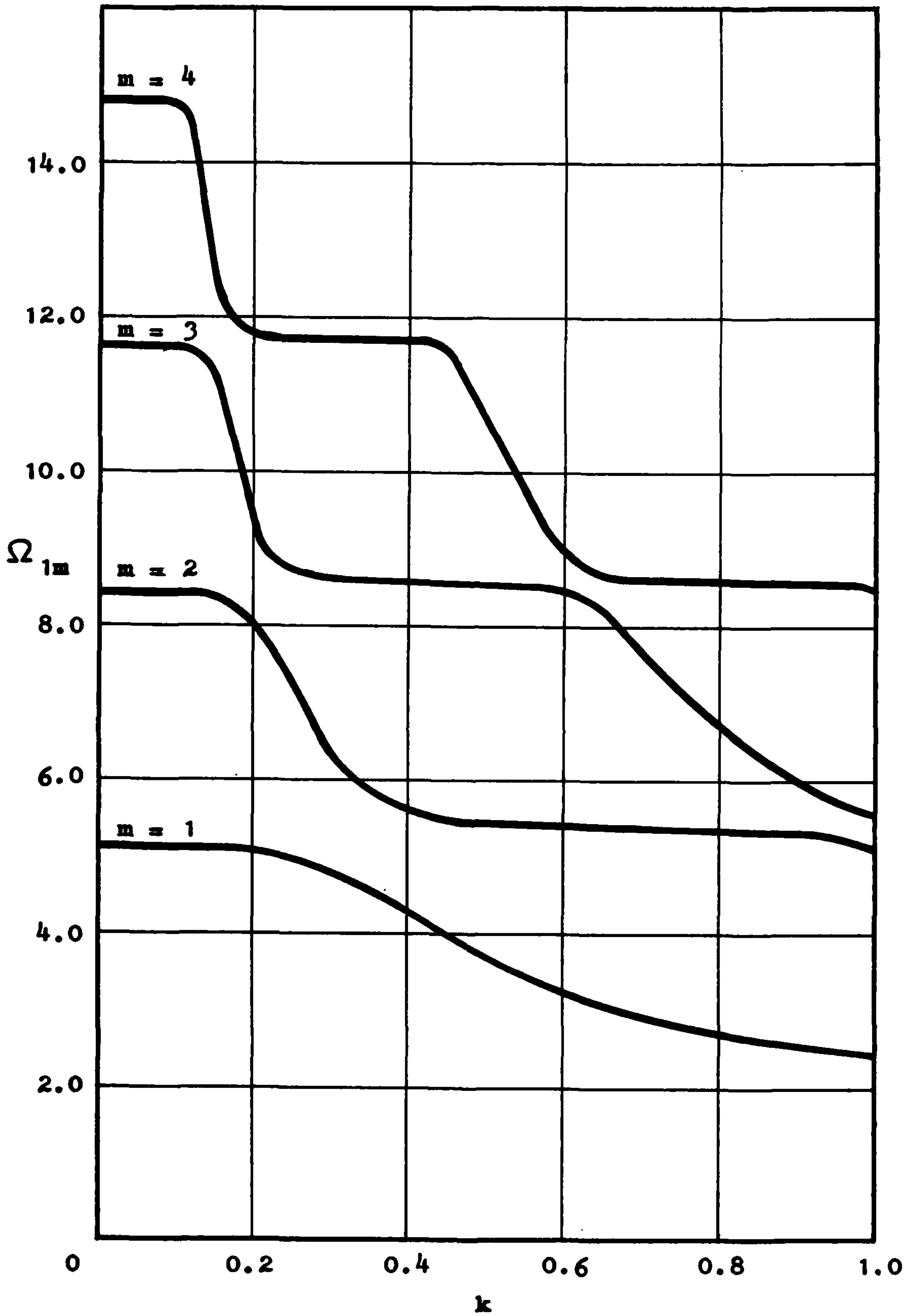

Fig. 1 Variation of the first-order (n = 1) circular frequency coefficients as functions of Poisson's ratio for free, transverse vibrations of a solid elastic core case-bonded to an infinitely-long, rigid, circular-cylindrical tank.

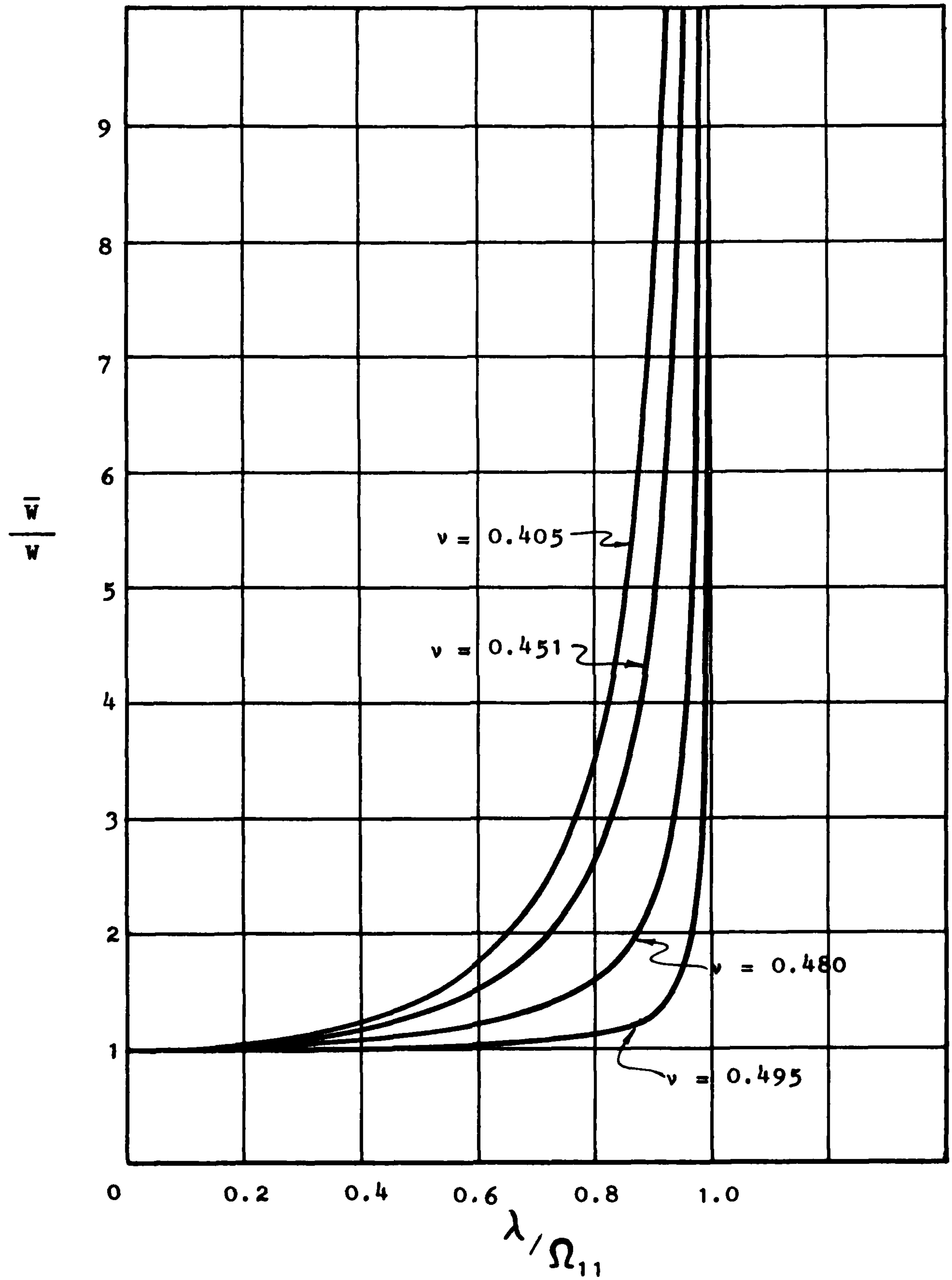

Figure 2(a) Amplitude ratio-frequency plots in the neighborhood of the fundamental resonance of a solid, elastic core case-bonded to an infinitely-long, rigid cylinder

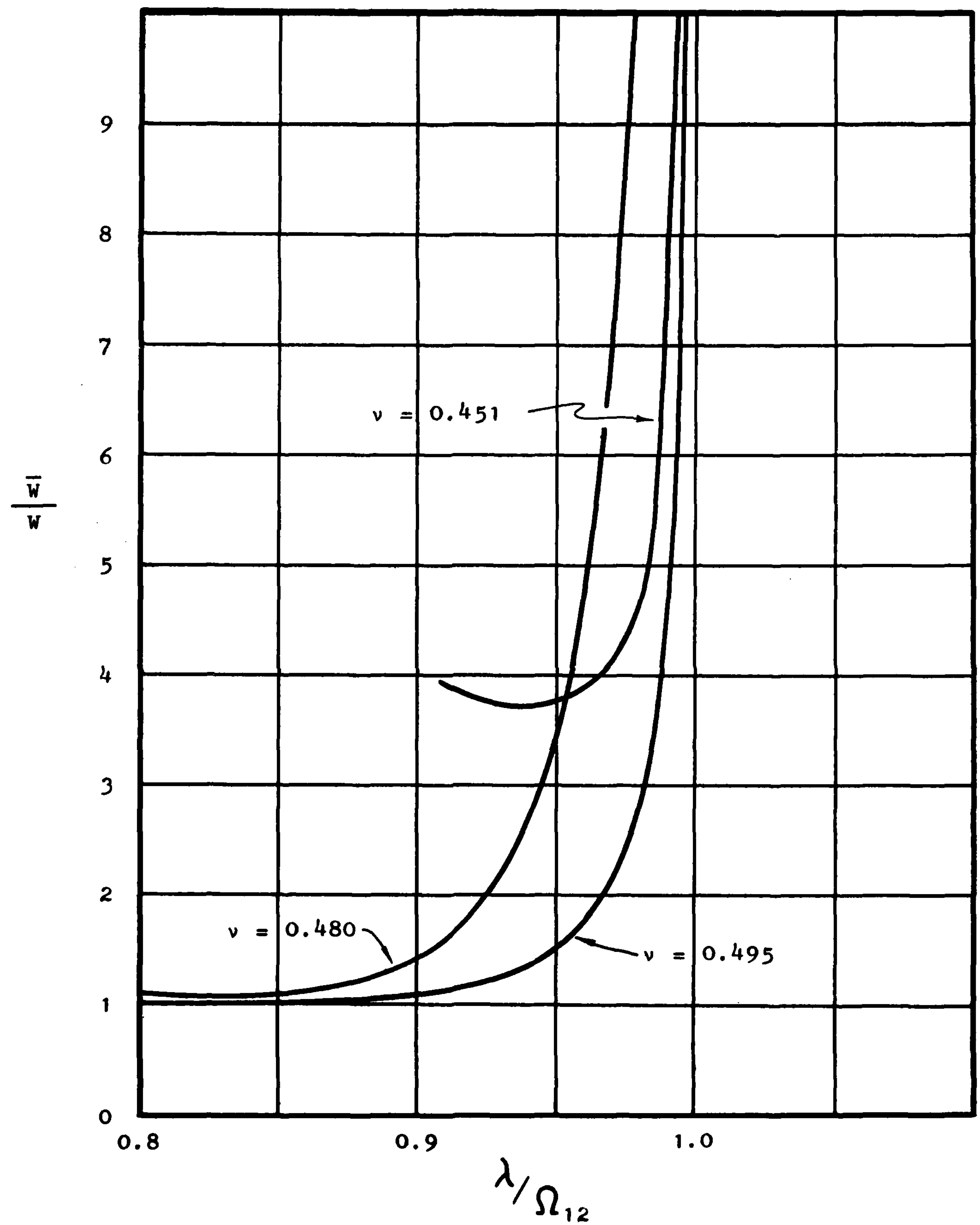

Figure 2(b) Amplitude ratio-frequency plots in the neighborhood of the second resonance for forced transverse vibrations of a solid, elastic core case-bonded to an infinitely-long, rigid cylinder

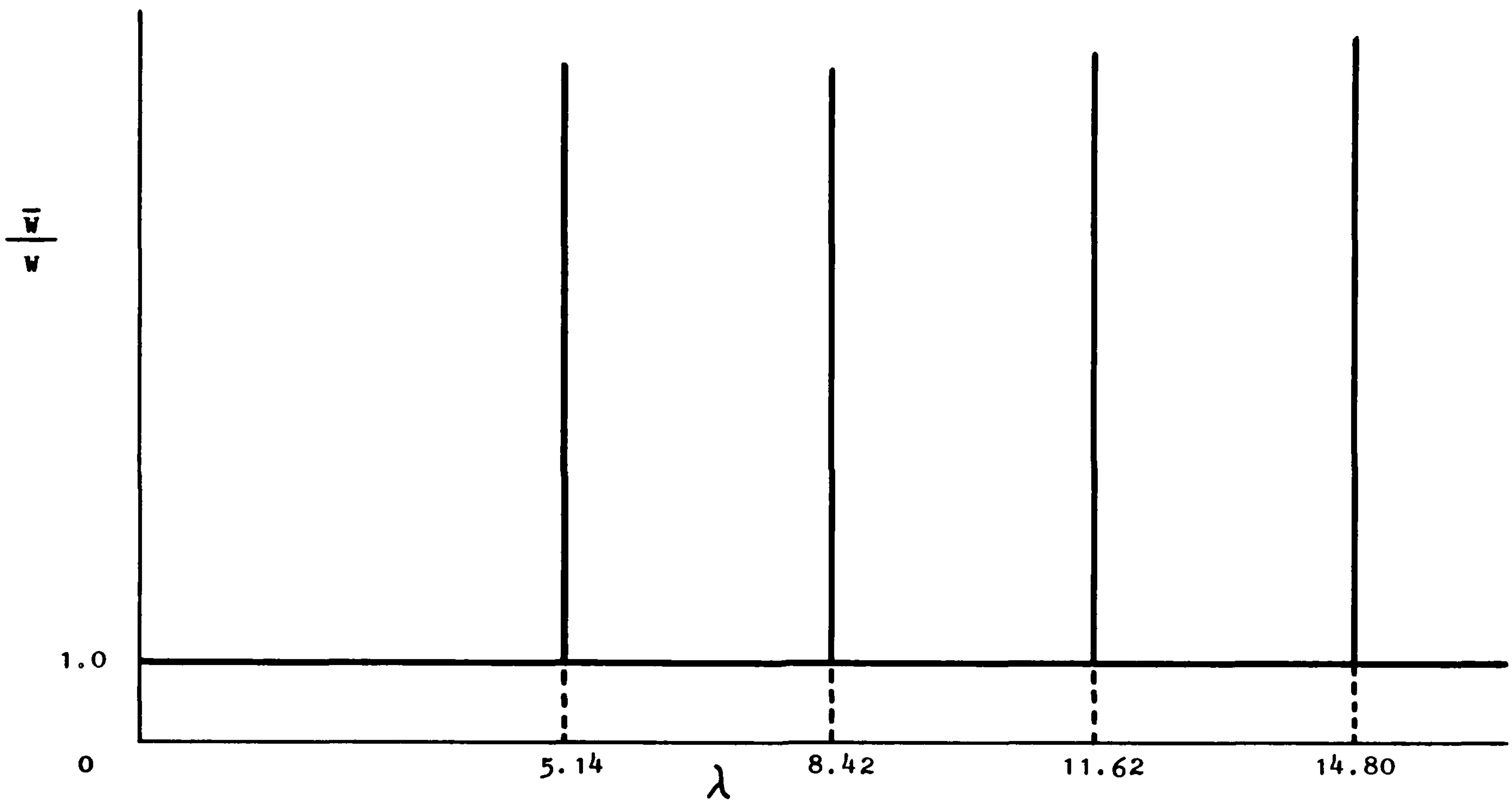

Figure 3 Amplitude ratio-frequency plot for forced transverse vibrations of a solid, incompressible elastic core case-bonded to an infinitely-long rigid cylinder

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